General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Wednesday 10 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

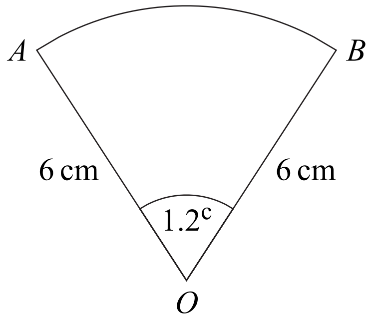
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle AOB is 1.2 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) Find the perimeter of the sector OAB. *(3 marks)*

2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x}\,\mathrm{d}x$$

giving your answer to three decimal places. *(4 marks)*

3 (a) Write down the values of p, q and r given that:

(i) $64 = 8^p$;

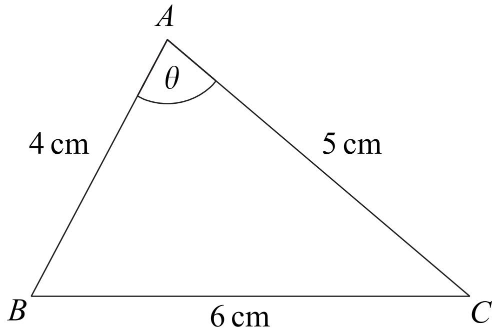
(ii) $\dfrac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of x for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$ *(2 marks)*

4 The triangle ABC, shown in the diagram, is such that $BC = 6\,\text{cm}$, $AC = 5\,\text{cm}$ and $AB = 4\,\text{cm}$. The angle BAC is θ.



(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(b) Hence use a trigonometrical identity to show that $\sin\theta = \frac{3\sqrt{7}}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

5 The second term of a geometric series is 48 and the fourth term is 3.

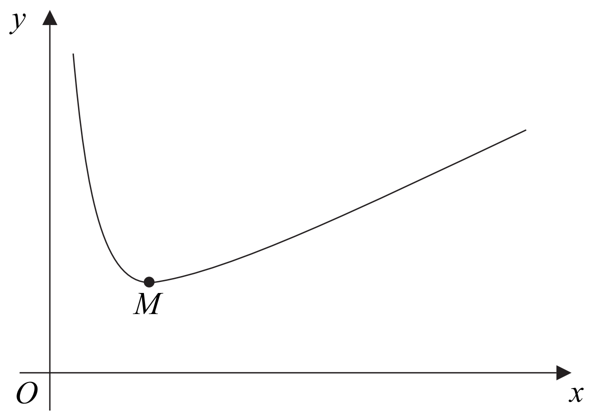
(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{4}$ and state the other value. *(4 marks)*

(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; *(1 mark)*

(ii) the sum to infinity of the series. *(2 marks)*

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \frac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \frac{4}{x^2}$, find $\frac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\int \left(x + 1 + \frac{4}{x^2} \right) dx$. *(3 marks)*

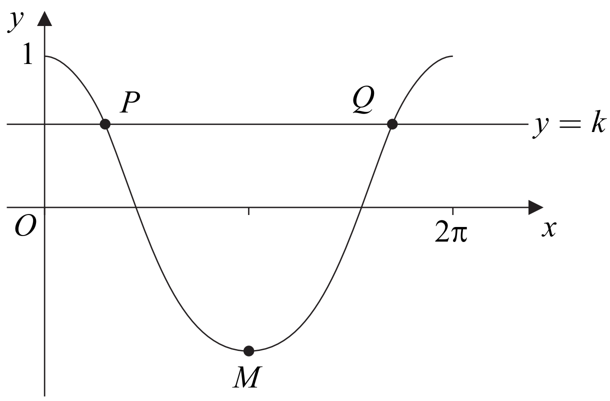
(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

7 (a) The first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$. *(3 marks)*

8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q. The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of Q in terms of π and α. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. *(2 marks)*

(d) Solve the equation $\cos 2x = \cos \dfrac{4\pi}{5}$ in the interval $0 \leqslant x \leqslant 2\pi$, giving the values of x in terms of π. *(4 marks)*

Turn over for the next question

9 (a) Solve the equation $3\log_a x = \log_a 8$. *(2 marks)*

(b) Show that

$$3\log_a 6 - \log_a 8 = \log_a 27$$

(3 marks)

(c) (i) The point $P(3, p)$ lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that $p = \log_{10}\left(\frac{27}{8}\right)$. *(2 marks)*

(ii) The point $Q(6, q)$ also lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that the gradient of the line PQ is $\log_{10} 2$. *(4 marks)*

END OF QUESTIONS

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